Sample Question Paper - 8
Mathematics-Basic (241)
Class- X, Session: 2021-22
TERM II

Time Allowed: 2 hours **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
3. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
4. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study-based questions.

Section A

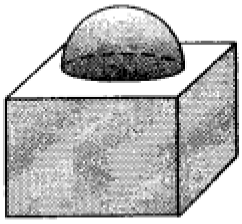
1. Solve the quadratic equation by factorization method: **[2]**

$a^2b^2x^2 + b^2x - a^2x - 1 = 0$

OR

Check the equation is quadratic equation or not: $(x - 2)(x + 1) = (x - 1)(x + 3)$

2. A solid is made up of a cube and a hemisphere attached on its top, as shown in the figure. Each edge of the cube measures 5 cm and the hemisphere has a diameter of 4.2 cm. Find the total area to be painted. [Take $\pi = \frac{22}{7}$] **[2]**



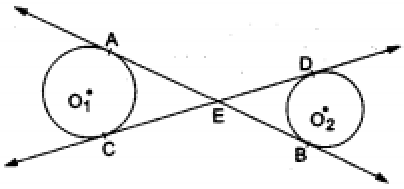
3. Find the missing frequency f for the following data , if mode for the following data is 39. **[2]**

Class	**5-15**	**15-25**	**25-35**	**35-45**	**45-55**	**55-65**	**65-75**
Frequency	2	3	f	7	4	2	2

4. In an A.P, if $S_n = n(4n + 1)$, find the A.P. **[2]**

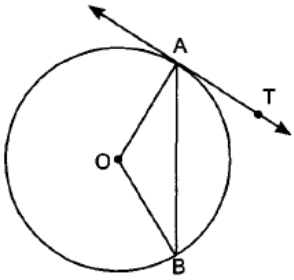
5. Find the combined mean of a group of 150, if the the value of mean of 50 students is 40 and that of other 100 students is 50. **[2]**

6. In the given figure, common tangents AB and CD to the two circles with centres O_1 and O_2 intersect at E. Prove that AB = CD. **[2]**



OR

In given figure, O is the centre of the circle, AB is a chord and AT is the tangent at A. If ∠AOB = 100° then find ∠BAT.



Section B

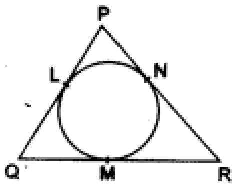
7. Find the sum of all two-digit natural numbers which are divisible by 4. **[3]**

8. The angle of elevation of a jet fighter from point A on ground is 60^0. After flying 10 seconds, the angle changes to 30°. If the jet is flying at a speed of 648 km/hour, find the constant height at which the jet is flying. **[3]**

OR

From a top of a building 100 m high the angle of depression of two objects are on the same side observed to be 45^0 and 60^0. Find the distance between the objects.

9. In the given figure, a circle is inscribed in a triangle PQR. If PQ = 10 cm, QR = 8 cm and PR = 12 cm, find the lengths of QM, RN and PL. **[3]**



10. Find the values of k for which the quadratic equation $(3k + 1)x^2 + 2(k + 1)x + 1 = 0$ has real and equal roots **[3]**

Section C

11. Draw a circle of radius 2 cm with centre O and take a point P outside the circle such that OP = 6.5 cm. From P, draw two tangents to the circle. **[4]**

OR

Draw a pair of tangents to a circle of radius 4.5 cm, which are inclined to each other at an angle of 45°.

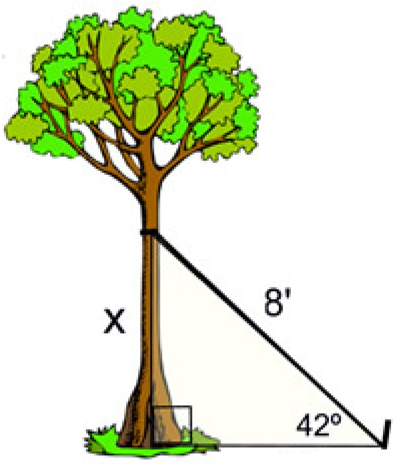
12. To find out the concentration of SO_2 in the air (in parts per million, i.e., ppm), the data was collected for 30 localities in a certain city and is presented below: **[4]**

Concentration of SO_2 (in ppm)	Frequency
0.00 - 0.04	4
0.04 - 0.08	9

0.08 - 0.12	9
0.12 - 0.16	2
0.16 - 0.20	4
0.20 - 0.24	2

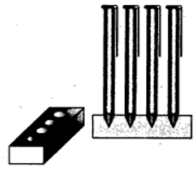
find the mean concentration of SO_2 in the air.

13. A nursery plants a new tree and attaches a guy wire to help support the tree while its roots take hold. An eight-foot wire is attached to the tree and to a stake in the ground. From the stake in the ground the angle of elevation of the connection with the tree is 42°. **[4]**



i. Find to the *nearest tenth of a foot*, the height of the connection point on the tree.
ii. If the angle of elevation changes to 30^0, keeping the height of the connection point on the tree same as calculated in (i), what should be the length of the wire?

14. A carpenter in the small town of Bareilly used to make and sell different kinds of wood items like a rectangular box, cylindrical pen stand, and cuboidal pen stand. One day a student came to his shop and asked him to make a pen stand with the dimensions as follows: **[4]**
A pen stand should be in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid should be 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm.



By using the above-given information, find the following:
i. The volume of the cuboid.
ii. The volume of wood in the entire stand.

Solution

MATHEMATICS BASIC 241

Class 10 - Mathematics

Section A

1. Given;

   $a^2b^2x^2 + b^2x - a^2x - 1 = 0$

   $\Rightarrow b^2x(a^2x + 1) - 1(a^2x + 1) = 0$

   $\Rightarrow (a^2x + 1)(b^2x - 1) = 0$

   $\Rightarrow x = \frac{-1}{a^2}$ or $x = \frac{1}{b^2}$

   OR

   The given equation is (x-2) (x+1) = (x-1) (x+3)

   $\Rightarrow x^2 - 2x + x - 2 = x^2 + 3x - x - 3$

   $\Rightarrow 3x - 1 = 0$

   Which is not of the form $ax^2 + bx + c = 0$, $a \neq 0$

   Hence, the given equation is not a quadratic equation.

2. According to the question, Edge of the cube(a) is = 5 cm.

   Diameter of the hemisphere is = 4.2 cm

   Radius of the hemisphere$(r) = \left(\frac{4.2}{2}\right)$ cm $= 2.1$cm

   Total area to be painted = total surface area of the cube - base area of the hemisphere + curved surface area of the hemisphere

   $= 6a^2 - \pi r^2 + 2\pi r^2$

   $= 6a^2 + \pi r^2$

   $= \left(6 \times 5 \times 5 + \frac{22}{7} \times 2.1 \times 2.1\right) \text{cm}^2$

   $= (150 + 13.86) \text{ cm}^2$

   $= 163.86 \text{ cm}^2$

3. Here , given mode is 39, which lies between 35-45 . Therefore, the modal class is 35-45.

   $l = 35, f_1 = 7, f_0 = f, f_2 = 4, h = 10$

   we know that, Mode = $l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

   $\Rightarrow 39 = 35 + \frac{7-f}{2(7)-f-4} \times 10$

   $\Rightarrow 4 = \frac{7-f}{2(7)-f-4} \times 10$

   $\Rightarrow 40 - 4f = 70 - 10f$

   $\Rightarrow 6f = 30$

   $\Rightarrow f = 5$

4. $S_n = n(4n + 1)$

   $S_{n-1} = (n - 1)[4(n - 1) - 1]$

   $= (n - 1)[4n - 5]$

   $S_n - S_{n-1} = n(4n + 1) - (n - 1)(4n - 5)$

   $(a_1 + a_2 + a_3 + --- + a_{n-1} + a_n) - (a_1 + a_2 + a_3 + --- + a_{n-1}) = 4n^2 + n - (4n^2 - 5n - 4n + 5)$

   $a_n = 10n - 5$

   For a_1,

   Put n = 1 so $a_1 = 10(1) - 5 = 5$

   For a_2,

   Put n = 2, so $a_2 = 10(2) - 5 = 15$

   For a_3,

   Put n = 3 so $a_3 = 10(3) - 5 = 25$

For a_4,

Put n = 4 so $a_1 = 10(4) - 5 = 35$

So A.P is 5, 15, 25, 35, ...

5. $n_1 + n_2 = 150$, $n_1 = 50$, $\overline{x}_1 = 40$, $\overline{x}_2 = 50$

$\overline{x} = \frac{n_1\overline{x}_1 + n_2\overline{x}_2}{n_1+n_2}$

$= \frac{50\times40+100\times50}{150}$

$= \frac{2000+5000}{150}$

$= \frac{7000}{150} = \frac{700}{15}$

= 46.666

= 46.67(approx.)

6. We know that tangent segments to a circle from the same external point are congruent.

So, $EA = EC$ for the circle having centre O_1

And, $ED = EB$ for the circle having centre O_2

Now, Adding ED on both sides in $EA = EC$, we get

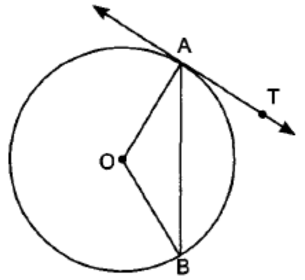
$EA + ED = EC + ED$

$\Rightarrow EA + EB = EC + ED$

$\Rightarrow AB = CD$

OR

Given, $\angle$AOB $=100^{o}$



Let $\angle$OAB = x = $\angle$OBA

$\angle$AOB + $\angle$OAB + $\angle$OBA = 180°

$\Rightarrow$ 100° + x + x = 180°

$\Rightarrow$ 2x = $180^{o} - 100^{o}$

$\Rightarrow$ 2x = 80°

$\Rightarrow$ x = 40°

$\angle$OAB + $\angle$BAT =90°

$40^\circ + \angle$BAT = 90°

Therefore, $\angle$BAT = 50°

Section B

7. All the two-digit natural numbers divisible by 4 are 12, 16, 20, 24, ..., 96

Here, $a_1 = 12$

$a_2 = 16$

$a_3 = 20$

$a_4 = 24$

$\therefore a_2 - a_1 = 16 - 12 = 4$

$a_3 - a_2 = 20 - 16 = 4$

$a_4 - a_3 = 24 - 20 = 4$

$\because a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \dots$ (= 4 each)

$\therefore$ This sequence is an arithmetic progression whose common difference is 4.

Here, a = 12, d = 4, l = 96

Let the number of terms be n.

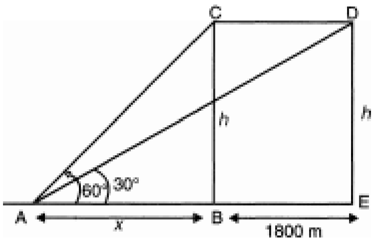
Then, $l = a + (n - 1)d \Rightarrow 96 = 12 + (n - 1)4$

$\Rightarrow$ 96 - 12 = (n - 1) 4 $\Rightarrow$ 84 = (n - 1)4

$\Rightarrow$ (n - 1) 4 = 84 $\Rightarrow$ n - 1 $= \frac{84}{4}$

$\Rightarrow$ n - 1 = 21 $\Rightarrow$ n = 21 + 1 $\Rightarrow$ n = 22

$\therefore S_n = \frac{n}{2}(a+l) = \frac{22}{2}(12+96) = (11)(108) = 1188$

8. 

1 hr =3600 sec

Hence in 3600 sec distance travelled by plane = 648 km = 648000 m

In 10 sec distance travelled by plane = $\frac{648000}{3600} \times 10$ = 1800 m

So BE = CD= 1800 m

In $\triangle$ABC,

$\frac{h}{x} = \tan 60^\circ$

$\frac{h}{x} = \sqrt{3}$

$\Rightarrow h = x\sqrt{3}$......(i)

In $\triangle$ADE we have

$\frac{h}{x+1800} = \tan 30^\circ$

$\frac{h}{x+1800} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad h = \frac{x+1800}{\sqrt{3}}$.........(ii)

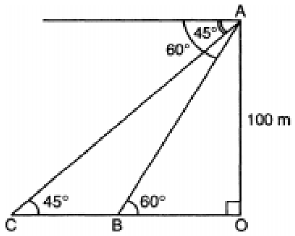
From equation (i) and (ii) we get

$x\sqrt{3} = \frac{x+1800}{\sqrt{3}}$

3x = x + 1800

x = 900 m So h = 900√3 meter

OR



In the given figure,

$\angle ACO = \angle CAX = 45^\circ$

and $\angle ABO = \angle XAB = 60^\circ$

Let A be a point and B, C be two objects.

In $\triangle$ AOC, $\frac{AO}{CO} = \tan 45^\circ$

$\Rightarrow \quad \frac{100}{\text{CO}} = 1$

$\Rightarrow CO = 100m$

Also in $\triangle$ ABO, $\frac{AO}{OB} = \tan 60^\circ$

$\Rightarrow \quad \frac{100}{OB} = \sqrt{3}$

$\Rightarrow \quad OB = \frac{100}{\sqrt{3}}$

$\therefore \quad BC = CO - OB = 100 - \frac{100}{\sqrt{3}}$

$= 100\left(1 - \frac{1}{\sqrt{3}}\right)$ m

$100\frac{(\sqrt{3}-1)}{\sqrt{3}} = 100\frac{(\sqrt{3}-1)}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

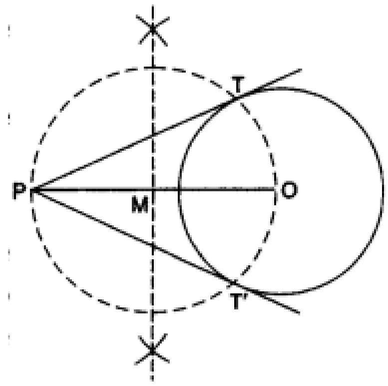
$= \frac{100(3-\sqrt{3})}{3}$m

9. According to question we are given that PQ = 10 cm, QR = 8 cm and PR = 12 cm.
We know that the lengths of the tangents drawn from an external point to a circle are equal.
Let PL = PN = x;
QL = QM = y;
RM = RN =z.
Now,PL + QL = PQ
$\Rightarrow$ x+y = 10, ...(i)
QM + RM = QR
$\Rightarrow$ y + z = 8, ...(ii)
Subtracting (ii) from (iii), we get
x - y = 4. ...(iv)
Solving (i) and (iv), we get
x = 7, y = 3.
Substituting y = 3 in (ii), we get z = 5
$\therefore$ QM = y = 3 cm,
RN = z = 5 cm,
PL = x = 7 cm.

10. The given equation is:
$(3k + 1)x^2 + 2(k + 1)x + 1 = 0$
This is of the form $ax^2 + bx + c = 0$, where
a = 3k + 1, b = 2(k + 1) = 2k + 2 and c = 1
As it is given that the given equation has real and equal roots, i.e., $D = b^2 - 4ac = 0$.
$\Rightarrow (2k + 2)^2 - 4(3k + 1) (1) = 0$
$\Rightarrow 4k^2 + 8k + 4 - 12k - 4 = 0$
$\Rightarrow 4k^2 - 4k = 0$
$\Rightarrow 4k(k - 1) = 0$
Therefore, either 4k = 0 or k - 1 = 0
$\Rightarrow$ k = 0 or k = 1
Hence, the roots of given equation are 1 and 0.

Section C

11. **STEPS OF CONSTRUCTION**
   1. Draw a circle of radius 2 cm and O as centre.
   2. Mark a point P outside the circle such that OP = 6.5 cm.

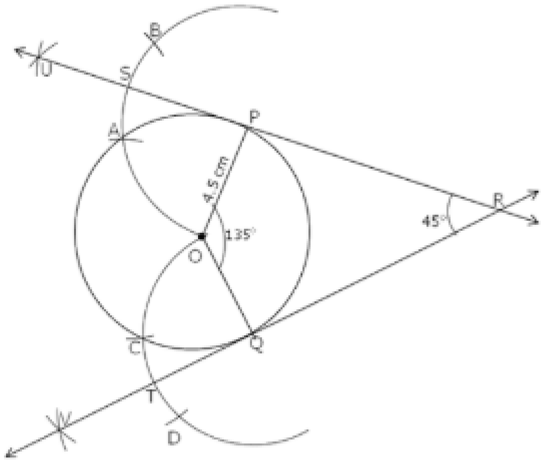


   3. Join OP and bisect it at M.
   4. Draw a circle with M as centre and radius equal to MP, to intersect the given circle at the points, T and T'.
   5. Join PT and PT'.

So, PT and PT' are required tangents.

OR

Steps of construction:-



i. Draw a circle having a centre O and a radius of 4.5 cm.
ii. Take point P on the circle and join OP.
iii. Angle between the tangents = 45°
Hence, the angle at the centre
$= 180^\circ - 45^\circ = 135^\circ$ (supplement of the angle between the tangents)
$\therefore$ Construct $\angle POQ = 135^\circ$
iv. Keeping a radius of 4.5 cm, draw arcs of circle taking the points P, and Q as the centres.
v. Name the points of intersection of arcs and circle as A and C respectively.
vi. Taking A as the centre and with the same radius mark B such that OA = AB.
vii. Similarly, taking C as the centre and with the same radius mark D such that OC = CD.
viii. Taking A and B as the centres and the same radius draw two arcs intersecting each other at U.
ix. Join P, S and U and extend it on both the sides to draw a tangent at point P.
x. Taking C and D as the centres and the same radius draw two arcs intersecting each other at V.
xi. Join Q, T and V and extend it on both the sides to draw a tangent at point Q.
xii. Extended tangents at P and Q intersect at R.
xiii. Hence, the required tangents are UR and VR such that the angle between them is 45° .

12. We may find class marks for each interval by using the relation
$x = \frac{\text{upperlimit+lowerclasslimit}}{2}$
Class size of this data = 0.04

Concentration of SO_2	**Frequency f_i**	**Class interval x_i**	**$d_i = x_i - 0.14$**	**u_i**	**f_iu_i**
0.00 – 0.04	4	0.02	-0.12	-3	-12
0.04 – 0.08	9	0.06	-0.08	-2	-18
0.08 – 0.12	9	0.10	-0.04	-1	-9
0.12 – 0.16	2	0.14	0	0	0
0.16 – 0.20	4	0.18	0.04	1	4
0.20 – 0.24	2	0.22	0.08	2	4
Total	$\sum f_i$ **= 30**				$\sum f_iu_i$**= -31**

let a = 0.14
Mean $\overline{x} = a + \left(\frac{\sum f_iu_i}{\sum f_i}\right) \times h$
= $0.14 + (0.04)\left(\frac{-31}{30}\right)$
= 0.099 ppm

13. i. According to the figure,
sin 42° = $\frac{x}{8}$

$0.669 = \frac{x}{8}$

x = 5.4'

So, height of the tree is 5.4 foot.

ii. Given: x = 5.4' and θ = 30^0

Let length of the wire be y foot.

Now, sin 30^0 = $\frac{5.4}{y}$

y = $\frac{5.4}{0.5}$ = 10.8

So, length of the wire is 10.8 foot.

14. i. Volume of the cuboid

$= 15 \times 10 \times 3.5 = 525 \text{cm}^3$

ii. Volume of a conical depression

$= \frac{1}{3}\pi(0.5)^2(1.4)$

$= \frac{1}{3} \times \frac{22}{7} \times 0.25 \times \frac{14}{10} = \frac{11}{30}\text{cm}^3$

$\therefore$ Volume of four conical depressions

$= 4 \times \frac{11}{30}\text{cm}^3 = \frac{22}{15}\text{cm}^3 = 1.47\text{cm}^3$

$\therefore$ Volume of the wood in the entire stand

$= 525 - 1.47 = 523.53\text{cm}^3$